TECHNICAL NOTE:

A SCOUR DEPTH APPROACH FOR DERIVING ERODIBILITY PARAMETERS FROM JET EROSION TESTS

E. R. Daly, G. A. Fox, A. T. Al-Madhhachi, R. B. Miller

ABSTRACT. *Typically the erosion rate of cohesive soils is modeled using the excess shear stress equation, which includes two soil parameters: the erodibility coefficient (k_d) and the critical shear stress (τ_c). A jet erosion test (JET) is a standardized method available for deriving the erodibility of cohesive soils. The JET data are typically analyzed using a Blaisdell solution approach. A second solution approach based on direct parameter optimization to the measured scour depth data has recently been proposed but with limited evaluation. Therefore, the objectives of this research were to: (1) develop a new spreadsheet tool that simultaneously solves for the erodibility parameters using both solution approaches, (2) evaluate the solutions in terms of their ability to predict the observed scour depth data, and (3) quantify differences in the predicted erodibility parameters from the two approaches. A series of JETs conducted across the Illinois River watershed in eastern Oklahoma were used to evaluate the performance of the spreadsheet and the solution methodologies. The new scour depth solution provided improved fits to the original scour depth data along with being more stable in converging to a solution as a function of the initial parameter estimates. The automated spreadsheet provides an easy-to-use tool for deriving erodibility parameters from JETs.*

Keywords. *Cohesive soil erosion, Critical shear stress, Erodibility, Jet erosion test, Scour depth.*

Streambank erosion is known to be a significant source of sediment in many impaired streams (Simon et al., 2000; Wilson et al., 2008; Fox and Wilson, 2010). Particle detachment models are often employed to predict rates of streambank erosion due to fluvial processes within a basin. Commonly, the erosion rate of cohesive streambanks is simulated using the excess shear stress equation (Partheniades, 1965; Hanson, 1990a, 1990b), which is defined as:

$$\varepsilon_r = k_d \left(\tau - \tau_c\right)^a \tag{1}$$

where ε_r is the erosion rate (cm s^{-1}), k_d is the erodibility coefficient (cm^3 N^{-1} s^{-1}), τ is the average hydraulic boundary shear stress (Pa), τ_c is the critical shear stress (Pa), and a is an empirical exponent commonly assumed to be unity (Hanson, 1990a, 1990b; Hanson and Cook, 2004). Using this model, erosion initiates once τ exceeds τ_c, and k_d defines the rate at which particles are detached after erosion is initiated.

Numerous studies have derived k_d and τ_c for cohesive soils using different techniques: large flumes (Hanson, 1990a; Hanson and Cook, 1997), small flumes (Briaud et al., 2001), laboratory hole erosion test (Wan and Fell, 2004), and a submerged jet test (Hanson and Cook, 1997; Mazurek, 2010; Marot et al., 2011; Al-Madhhachi et al., 2013a, 2013b, 2013c). The submerged jet erosion test (JET) was developed for measuring these parameters *in situ* as well as in the laboratory (Hanson, 1990b; Hanson and Cook, 1997; Hanson and Simon, 2001). The JET device consists of an impinging jet connected to a constant water source, a "can" that serves to both hold the JET in position and to submerge the test soil in water, and a point gauge to measure the depth of scour produced by the JET. A detailed description of the JET and the testing methodology has been presented by numerous studies (Hanson and Cook, 1997; Hanson and Simon, 2001; Al-Madhhachi et al., 2013a).

Hanson and Cook (1997) and Hanson et al. (2002) developed the analytical methods to directly estimate k_d and τ_c based on diffusion principles using an Excel spreadsheet routine. The analytical methods were based on diffusion principles developed by Stein and Nett (1997). The rate of variation in the depth of scour was assumed to be the erosion rate as a function of the maximum stress at the boundary. The maximum shear stress was based on determining the diameter of the jet nozzle and the distance from the jet origin to the initial cohesive soil surface. Accordingly, τ_c was assumed to occur when the rate of scour was equal to zero at the equilibrium depth. Blaisdell et al. (1981) developed a hyperbolic function for predicting the equilibrium depth, which was used in the spreadsheet to calculate τ_c. The k_d was then determined depending on the measured

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scour depth, time, predetermined τ_c, and a dimensionless time function (Hanson et al., 2002).

Several flume studies have been conducted to measure the erosion of cohesive soils in order to verify the use of the JET (Hanson, 1990a; Hanson and Cook, 1997; Hanson and Simon, 2001). Hanson (1990a) measured soil erodibility in large outdoor channels with soil material placed throughout the entire length of the channel beds. Six channels were constructed (0.91 m wide and 30.5 m long) with different slopes: 0.5%, 1.5%, and 3%. Hanson (1990b) empirically related JET index values determined from the three soils to the soil erodibility values determined from the flume studies of Hanson (1990a). Hanson and Cook (1999) performed two open-channel flow tests in a large outdoor open channel (1.8 m wide and 29 m long with 2.4 m sidewalls) on compacted samples of lean clay and silty clay. The k_d and τ_c determined from those flume tests verified the use of *in situ* and laboratory JET experiments. This study as well as other studies (Hanson et al., 2002; Hanson and Cook, 2004) have verified the use of the JET to predict the rates of erosion for headcut migration, impinging jet scour, and embankment breach formation and widening.

In addition to the original JET, a new miniature version of the JET device, which is referred to as the "mini" JET, was recently developed by Hanson (Al-Madhhachi et al., 2013a). The "mini" JET device is smaller (975 cm^3) and lighter (4.2 kg) than the original JET device (28,130 cm^3 and 12.6 kg) and thus can be more easily handled in the field as well as in the laboratory (Al-Madhhachi et al., 2013a). The "mini" JET device was first used by Simon et al. (2010) in the field, where they performed 279 tests using the "mini" JET to measure k_d and τ_c utilizing the methods of testing and analysis developed for the original JET. They compared the "mini" JET results with the original JET device at 35 sites in the Tualatin River basin, Oregon, and observed good agreement in derived values of τ_c but observed differences in k_d and the k_d-τ_c relationships between the two JET devices (Simon et al., 2010). They hypothesized that the observed differences in results were due to differences in the size of the submergence cans between the original and "mini" JET devices. These tests were conducted *in situ* at side-by-side locations, but Al-Madhhachi et al. (2013a) hypothesized that the results were likely influenced by *in situ* heterogeneity and possible differences in methodology and setup.

Al-Madhhachi et al. (2013a) compared measured excess shear stress parameters using the two JET devices in a more controlled laboratory setting using two cohesive soils (clayey sand and silty sand). Statistically equivalent k_d values were derived by the two JET devices for both soils based on Mann-Whitney rank sum tests, but the τ_c values derived by the "mini" JET were consistently lower. Al-Madhhachi et al. (2013a) hypothesized that the measured differences in τ_c were due to the relative scale of the two submerged jets in comparison to the inherent soil structure created by the compaction method. Adjusting the equilibrium depth of the "mini" JET by a coefficient in the analysis resulted in insignificant differences in the estimated τ_c between the two JET devices. Al-Madhhachi et al. (2013a) concluded that the "mini" JET measurements, based on the excess stress model parameters, provided erosion rate predictions equivalent to the original JET. Al-Madhhachi et al. (2013b, 2013c) compared both the *in situ* original and "mini" JET devices with flume tests to predict soil erodibility on two cohesive soils. With these modifications, they concluded that the flume and both JET devices provided statistically equivalent soil erodibility estimates.

In order to estimate k_d as a function of τ_c for cohesive soils, Hanson and Simon (2001) suggested an inverse relationship between k_d and τ_c:

$$k_d = 0.2\tau_c^{-0.5} \tag{2}$$

Hanson and Simon (2001) derived their relationship based on 83 *in situ* JETs conducted on cohesive streambeds in the Midwestern U.S. A wide data range was observed, with τ_c spanning six orders of magnitude and k_d spanning four orders of magnitude. A general inverse relationship was observed between τ_c and k_d, suggesting that soils with a low τ_c have a high k_d and vice versa. Their relationship predicted the data with a coefficient of determination (R^2) of 0.64 and was incorporated into streambank erosion and stability models, such as the Bank Stability and Toe Erosion Model (BSTEM), as a tool for estimating k_d from τ_c (Midgley et al., 2012). This relationship was recently updated by Simon et al. (2011) based on hundreds of JETs on streambanks across the U.S.:

$$k_d = 1.62\tau_c^{-0.838} \tag{3}$$

In many cases, it has been reported that the Blaisdell equilibrium scour depth solution approach that forms the basis for deriving erodibility parameters does not always converge to a reasonable solution (Simon et al., 2010). A second solution approach based on direct parameter optimization to the measured scour depth data has recently been proposed by Robert Thomas (Department of Geography, University of Hull, U.K.) but with limited evaluation (Simon et al., 2010; Cossette et al., 2012; Daly et al., 2013). In fact, such an iterative solution was originally proposed by Hanson and Cook (1997) as "method 1," but the solver routine never converged to a stable solution and was therefore not investigated further in that paper. Simon et al. (2010) found that a solution methodology based on "method 1" provided a reduction in the scatter of the k_d-τ_c relationship, but the values obtained led to an overprediction of erosion when used in model simulations, while the original Blaisdell solution underpredicted erosion. Cossette et al. (2012) evaluated this optimized solution methodology along with the original methodology of Hanson and Cook (2004), a visual assessment methodology, and an equilibrium state methodology. The four methodologies predicted different critical shear stress values, although the relative ranking between different soils tested was consistent. They concluded that there is a need for a review of the theoretical framework of the JET and its underlying assumptions. While the Blaisdell solution methodology continues to be the default method for analyzing JET data at the present time, these current research studies have raised questions about the accuracy of values obtained from this analysis. Therefore, the objectives of this research were to: (1) develop a new spreadsheet tool that simultaneously solves for the erodibility parameters using two solution approaches, (2) evaluate the solution methodologies in terms of their ability to predict the observed scour depth data, and (3) quantify differences in the predicted erodibility parameters from the two approaches. This research utilized a series of JETs conducted across the Illinois River watershed in eastern Oklahoma.

MATERIALS AND METHODS

SOLUTION TECHNIQUES

Analytical methods for the JET were first presented by Hanson and Cook (1997, 2004), assuming that the rate of variation in the depth of scour (dJ/dt) was the erosion rate as a function of the maximum stress at the boundary, which was determined by the diameter of the jet nozzle and the distance from jet origin to the initial channel bed. Therefore, the erosion rate equation for jet scour is written as (Hanson and Cook, 1997):

$$\frac{dJ}{dt} = k_d\left[\frac{\tau_o J_p{}^2}{J^2} - \tau_c\right] \text{ for } J >= J_p \tag{4}$$

where J is the scour depth (cm), and J_p is the potential core length from jet origin (cm). Accordingly, τ_c was assumed to occur when the rate of scour was equal to zero at the equilibrium scour depth (J_e):

$$\tau_c = \tau_o\left(\frac{J_p}{J_e}\right)^2 \tag{5}$$

where $\tau_o = C_f \rho_w U_o^2$ is the maximum shear stress due to the jet velocity at the nozzle (Pa), $C_f = 0.00416$ is the coefficient of friction, ρ_w is water density (kg m^{-3}), U_o is the jet velocity at the orifice (cm s^{-1}), $J_p = C_d d_o$, d_o is the nozzle diameter (cm), and $C_d = 6.3$ is the diffusion constant. Equations 4 and 5 can be incorporated in a dimensionless form as the following equation:

$$\frac{dJ^*}{dT^*} = \frac{\left(1 - J^{*2}\right)}{J^{*2}} \tag{6}$$

where $J^* = J/J_e$ and $J_p{}^* = J_p/J_e$. Stein and Nett (1997) presented the reference time (T_r) as follows:

$$T_r = \frac{J_e}{k_d \tau_c} \tag{7}$$

and the dimensional time (T^*) was given as:

$$T^* = t \,/\, T_r \tag{8}$$

where t is the time of a data reading or scour depth measurement.

Equation 6 refers to the change in scour depth with time, for time T^*. Integration of equation 6 gives the following equation:

$$T^* - T_p{}^* = -J^* + 0.5\ln\left(\frac{1+J^*}{1-J^*}\right) + J_p{}^* - 0.5\ln\left(\frac{1+J_p{}^*}{1-J_p{}^*}\right) \tag{9}$$

The Excel spreadsheet discussed by Hanson and Cook (2004) used equations 4 through 9 to determine τ_c and k_d. The critical stress (τ_c) was determined from equation 5 based on the equilibrium scour depth (J_e). Blaisdell et al. (1981) noted that it was difficult to determine the equilibrium scour depth due to the large time required to reach J_e. Therefore, the spreadsheet calculated the equilibrium scour depth using the scour depth data versus time and a hyperbolic function for determining the equilibrium scour depth developed by Blaisdell et al. (1981). The general form of this equation is:

$$(f - f_o)^2 - x^2 = A_1{}^2 \tag{10}$$

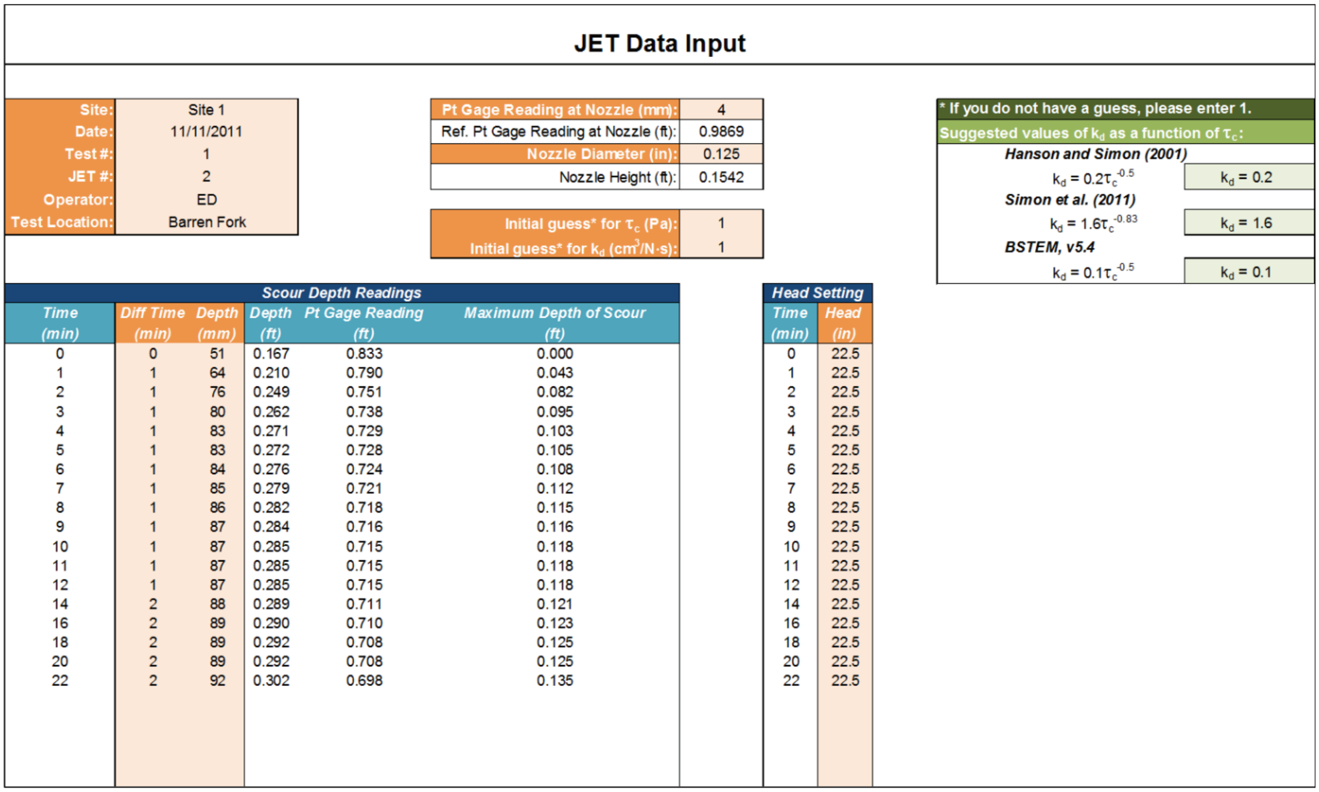
where A_1 is the value for the semi-transfer and semi-conjugate of the hyperbola, $f = \log(J/d_o) - x$, $x = \log[(U_o t)/d_o]$, and $f_o = \log(J_e/d_o)$. From fitting the scour depth data based on plotting f versus x, the coefficients A_1 and f_o can be determined using Microsoft Excel Solver, and then J_e can be determined ($J_e = d_o 10^{f_o}$). The spreadsheet was then used to calculate k_d by fitting the curve of measured data based on equation 9. The k_d depends on the measured scour depth, time, pre-estimated τ_c, and the dimensional time function (Hanson et al., 2002).

A second solution of the excess shear stress equation has been proposed by multiple researchers (Simon et al., 2010; Daly et al., 2013). The proposed alternative plotted the original scour depth versus time as derived from the JETs. Then, using the applied shear stress and the initial parameter estimates, k_d and τ_c were fit to the observed scour depth data using the solver routine in Microsoft Excel (generalized reduced gradient method) to minimize the sum of squared errors between the measured scour data and the solution of the excess shear stress equation. This procedure mimics the approach used by Al-Madhhachi et al. (2013b, 2013c) for a mechanistic detachment model.

While this solution approach has been proposed previously by Hanson and Cook (1997), it was originally found to be unstable, as it allowed for multiple solutions depending on the initial iteration values, and therefore was neglected in favor of the Blaisdell solution. In order to check convergence, the scour depth and Blaisdell solution approaches were both tested using a series of initial guesses for the k_d and τ_c values. Various initial values of τ_c were selected with the corresponding initial value of k_d determined using the Simon et al. (2011) relationship shown in equation 3.

JET SPREADSHEET

To incorporate the recently proposed scour depth solution approach, an automated spreadsheet routine has been created following the original spreadsheet routine developed by Hanson and Cook (2004). This updated routine includes both the Blaisdell solution as well as the scour depth solution approach. The Data Input sheet allows the user to directly input field data from the JET without con-



JET Data Input

Site:	Site 1
Date:	11/11/2011
Test #:	1
JET #:	2
Operator:	ED
Test Location:	Barren Fork

Pt Gage Reading at Nozzle (mm):	4
Ref. Pt Gage Reading at Nozzle (ft):	0.9869
Nozzle Diameter (in):	0.125
Nozzle Height (ft):	0.1542

Initial guess* for τ_c (Pa):	1
Initial guess* for k_d ($cm^3/N \cdot s$):	1

* If you do not have a guess, please enter 1.

Suggested values of k_d as a function of τ_c:

Hanson and Simon (2001)	
$k_d = 0.2\tau_c^{-0.5}$	$k_d = 0.2$
Simon et al. (2011)	
$k_d = 1.6\tau_c^{-0.83}$	$k_d = 1.6$
BSTEM, v5.4	
$k_d = 0.1\tau_c^{-0.5}$	$k_d = 0.1$

Scour Depth Readings

Time (min)	Diff Time (min)	Depth (mm)	Depth (ft)	Pt Gage Reading (ft)	Maximum Depth of Scour (ft)
0	0	51	0.167	0.833	0.000
1	1	64	0.210	0.790	0.043
2	1	76	0.249	0.751	0.082
3	1	80	0.262	0.738	0.095
4	1	83	0.271	0.729	0.103
5	1	83	0.272	0.728	0.105
6	1	84	0.276	0.724	0.108
7	1	85	0.279	0.721	0.112
8	1	86	0.282	0.718	0.115
9	1	87	0.284	0.716	0.116
10	1	87	0.285	0.715	0.118
11	1	87	0.285	0.715	0.118
12	1	87	0.285	0.715	0.118
14	2	88	0.289	0.711	0.121
16	2	89	0.290	0.710	0.123
18	2	89	0.292	0.708	0.125
20	2	89	0.292	0.708	0.125
22	2	92	0.302	0.698	0.135

Head Setting

Time (min)	Head (in)
0	22.5
1	22.5
2	22.5
3	22.5
4	22.5
5	22.5
6	22.5
7	22.5
8	22.5
9	22.5
10	22.5
11	22.5
12	22.5
14	22.5
16	22.5
18	22.5
20	22.5
22	22.5

Figure 1. Example of the Data Input sheet from the updated spreadsheet routine. Required input data are highlighted in orange.

version factors (fig. 1). The required input includes the time between readings, the point gauge readings, the head setting, the point gauge reading at the nozzle, the nozzle diameter, and initial parameter estimates for τ_c and k_d (fig. 1). If the user does not have an initial estimate, a value of 1 may be entered for both parameters, or the suggested values of k_d as a function of τ_c may be used (Hanson and Simon, 2001; Simon et al., 2011). These initial parameter estimates are utilized to aid in solution convergence using the generalized reduced gradient method solver routine.

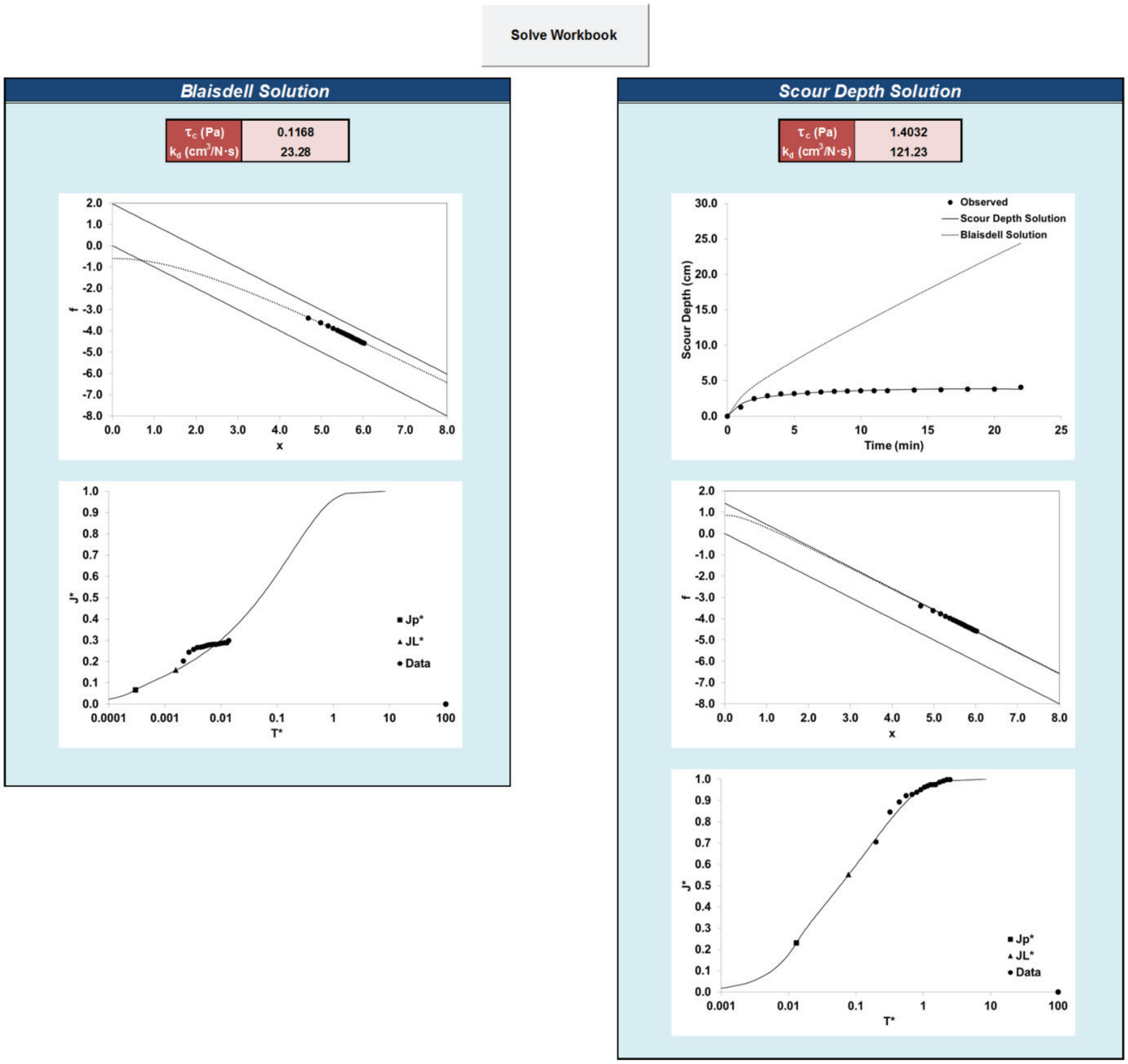
After the user has input all required data, the Solve tab is used. A button at the top of the worksheet labeled "Solve Workbook" activates the automated routine. Solver routines for both solution approaches are iteratively performed three times to ensure convergence. The routine first estimates the erodibility parameters using the Blaisdell solution following the original methodology as outlined by Hanson and Cook (2004). The results of this solution are shown in the box labeled "Blaisdell Solution" (fig. 2). The routine then derives erodibility parameters using the scour depth solution and reports its results in the box labeled "Scour Depth Solution" (fig. 2). After the scour depth solution approach is completed, the routine back-calculates the Blaisdell solution with the new τ_c and k_d solutions by updating the equilibrium scour depth (J_e) and the parameter f_o. With a fixed f_o and J_e, A is solved for using the solver routine in Microsoft Excel (generalized reduced gradient method). From here, an updated J^* and T^* are calculated and displayed as the dimensionless scour function optimization. For comparison, the dimensionless scour function optimization plot is shown for both the Blaisdell solution and the scour depth solution (fig. 3). Also for comparison, the observed and predicted scour depths are plotted and displayed on the Solve sheet (fig. 4).

FIELD DATA

The *in situ* JETs were performed on streambanks in the Illinois River basin in northeastern Oklahoma, one of the state's high-priority basins. The basin falls within the Ozark Highlands ecoregion, which typically contains streams that are riffle and pool dominated, clear, and have coarse gravel, cobble, or bedrock substrates. Banks are typically composite and include a silty loam top layer with an unconsolidated gravel bottom layer and toe (Fuchs et al., 2009; Fox et al., 2011; Heeren et al., 2012; Midgley et al., 2012).

Detailed stream reach data were collected at 13 sites within the Illinois River basin (fig. 5). Sites were distributed over a variety of stream orders. Locations for data collection were chosen based on accessibility. Data collection at each site included soil samples from the cohesive layers of the streambank and "mini" JETs along a representative stream reach (Al-Madhhachi et al., 2013a, 2013b, 2013c). At least one JET was performed (fig. 6) *in situ* at each site where the streambanks had a cohesive soil layer. The "mini" JETs were set up and operated following procedures outlined by Al-Madhhachi et al. (2013a, 2013b).

The parameters derived from each solution approach were used to predict the scour depth over time within the excess shear stress equation. The normalized objective function (NOF) (Fox et al., 2006; Al-Madhhachi et al., 2013a, 2013b) was calculated to quantify the goodness of fit. The NOF is the ratio of the standard deviation (STDD) of differences between observed and predicted data to the overall mean (X_a) of the observed data:



Blaisdell Solution

τ_c (Pa)	0.1168
k_d (cm^3/N·s)	23.28

Scour Depth Solution

τ_c (Pa)	1.4032
k_d (cm^3/N·s)	121.23

Figure 2. Example of the Solve sheet from the updated spreadsheet routine.

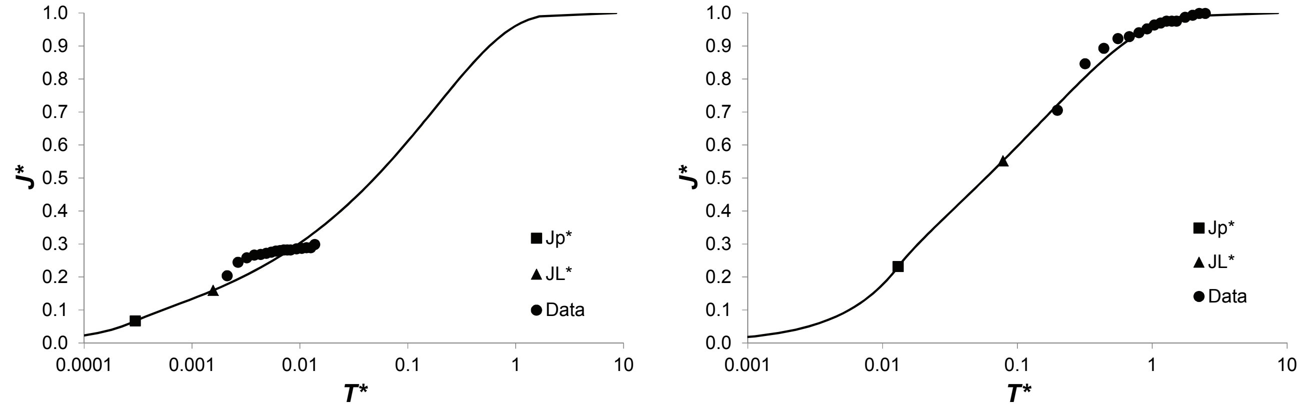


Figure 3. Example dimensionless scour function optimization using the Blaisdell solution (top) and the scour depth solution (bottom). J^* is dimensionless scour depth, and T^* is dimensionless time.

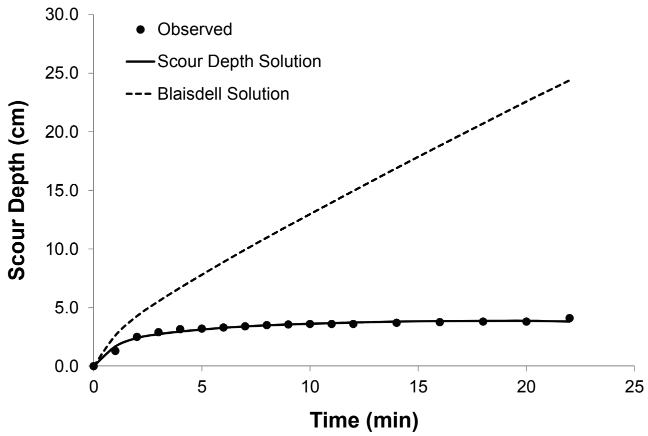


Figure 4. Example of observed and predicted scour depths using the Blaisdell solution and scour depth solution.

$$\mathrm{NOF}=\frac{\mathrm{STDD}}{X_a}=\frac{\sqrt{\dfrac{\sum_{i=1}^{N}(x_i-y_i)^2}{N}}}{X_a} \tag{11}$$

where x_i and y_i are the observed and predicted data, respectively, and N is the number of observations. In general, 1%, 10%, and 100% deviations from the observed values result in NOF values of 0.01, 0.1, and 1.0, respectively (Fox et al., 2006).

RESULTS AND DISCUSSION

For most of the in-field JETs, the original spreadsheet routine proposed by Hanson and Cook (2004) did not always converge to a reasonable solution based on a visual observation of the J^* versus T^* plot (see fig. 3 as an example). The scour depth solution provided much improved fits of J^* versus T^* (fig. 3) and therefore much improved fits (lower NOF values) to the original scour depth data measured during the JET (table 1). Typically, the Hanson and Cook (2004) approach resulted in a lower τ_c and a corresponding k_d that resulted in an overestimation of the scour depth over time (table 1). An analytical method that overpredicts scour depth may be viewed as a conservative approach from a design standpoint, but it poses drawbacks for testing and understanding erosion processes.

The scour depth solution was found to be stable regardless of the initial parameter estimates (table 2). This analysis was performed on a JET test from two of the 13 sites representing a range of expected τ_c and k_d values. For Barren Fork Site 1 (the more erodible case, with slightly cohesive silt loam soil), both the Blaisdell solution and the scour depth solution converged on the same parameter values each time, although unique from each other (table 2). For Barren Fork Site 2 (the less erodible case, with cemented silt loam soil), the scour depth solution was found to be stable, converging on the same answer with each initial parameter estimate; however, the Blaisdell solution was not always able to converge on a solution for k_d, specifically for the lower initial τ_c values (table 2).

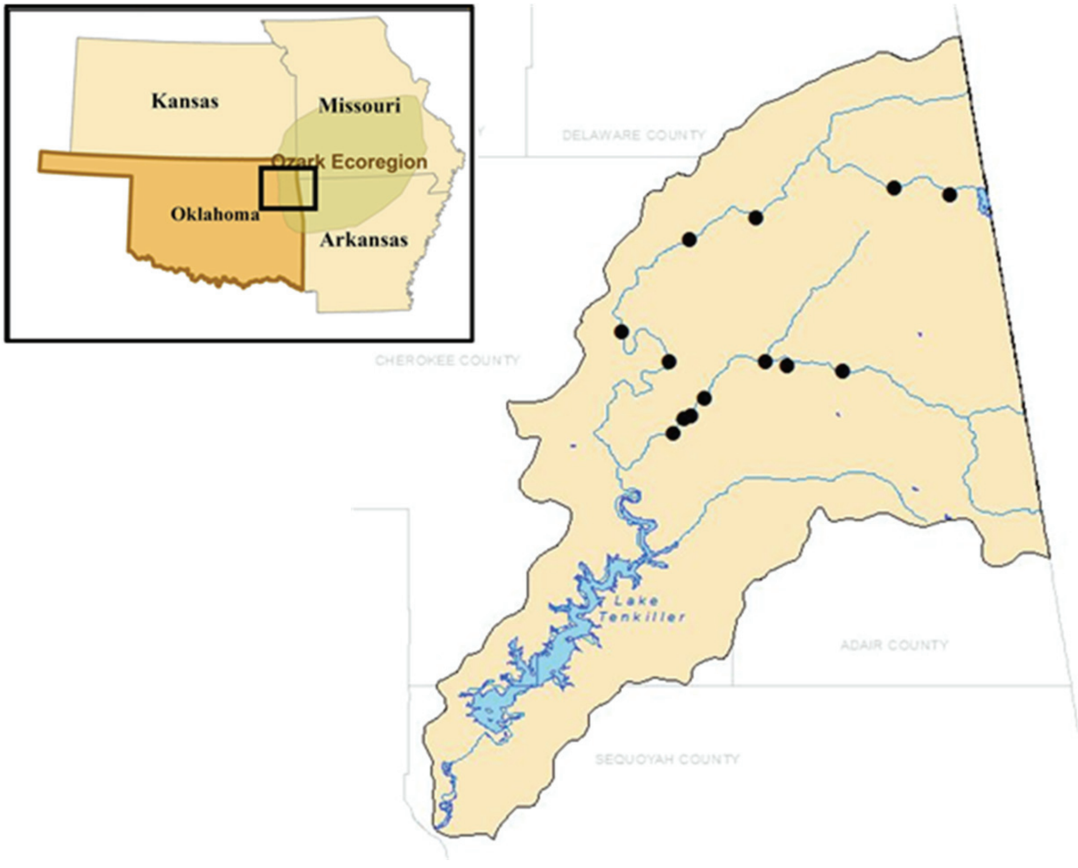


Figure 5. The Illinois River basin (Oklahoma only) with 13 sites (circles) at which JETs were conducted.

Figure 6. Example of “mini” JET being performed (top) and example of typical bank profile (bottom).

Table 1. Solutions based on varying initial guesses of τ_c and k_d for the Blaisdell solution and the scour depth solution. See figure 4 for an example solution for both approaches.

	Blaisdell Solution (Hanson and Cook, 2004)			Scour Depth Solution		
Site	k_d (cm^3 N^{-1} s^{-1})	τ_c (Pa)	NOF	k_d (cm^3 N^{-1} s^{-1})	τ_c (Pa)	NOF
1	23.3	0.1	3.4[a]	121.2	1.4	0.1
2	13.2	<0.1	0.2	12.1	<0.1	0.2
3	4.4	0.4	4.6[a]	27.5	2.8	0.1
4	3.6	0.0	0.8[a]	6.6	3.1	0.1
5	1.1	1.3	3.3[a]	6.6	14.3	0.1
6	4.1	0.1	2.5[a]	17.6	3.1	0.1
7	28.6	0.3	5.3[a]	210.8	1.2	0.1
8	1.4	0.0	0.5[a]	2.0	2.8	0.2
9	7.7	0.4	4.6[a]	50.8	2.5	0.1
10	22.1	1.3	5.8[a]	194.1	2.3	0.1
11	11.0	0.5	4.8[a]	74.7	2.2	0.1
12	0.3	1.3	1.9[a]	0.9	16.4	0.1
13	3.1	0.9	5.4[a]	21.9	5.6	0.1

[a] Solution overpredicted the observed scour depth data, similar to the example in figure 4.

Using the scour depth solutions for this limited data set, an inverse relationship was observed between the two erodibility parameters (k_d and τ_c) in the excess shear stress equation (fig. 7). A power law relationship estimated k_d as a function of τ_c with $R^2 = 0.56$ for the parameters derived from the scour depth solution approach. Note that the updated Simon et al. (2011) relationship adequately predicted the k_d-τ_c relationship when the parameters were derived with the Blaisdell solution approach. While there was a similar trend between the measured data and the Hanson and Simon (2001) relationship for the k_d-τ_c relationship when derived from the scour depth approach, k_d calculated using the Hanson and Simon (2001) relationship would have been underestimated. Erosion rate predictions would consequently have been underestimated as well. With the new scour depth solution approach, previous relationships may need to be revisited.

CONCLUSIONS

The routines in the original JET spreadsheet did not always converge to a reasonable solution based on a visual observation of the dimensionless scour versus time. Therefore, a new spreadsheet tool has been developed to incorporate an automated scour depth solution approach similar to that proposed by Hanson and Cook (1997) and Simon et al. (2010). This tool provides both the Blaisdell solution and the scour depth solution approaches for use at the discretion of the user. The scour depth solution was stable within the ranges tested and converged on the same solution despite different initial parameter estimates. The scour depth solution fit the dimensionless scour function optimization better than the original Blaisdell solution, which tended to underpredict the critical shear stress. With the corresponding erodibility coefficient, the Blaisdell solution overpredicted the resulting scour depth, potentially resulting in a conservative design approach. Overprediction of scour depth may be valuable in situations such as dam construction where a large factor of safety is an engineering requirement, but a solution that more accurately represents the physical properties of the soil is preferable from a scientific and engineering standpoint. Results from the new scour depth solution showed similar trends in relationships between erodibility parameters as reported by previous research; however, these trends may need to be revisited with the alternative solution approach.

Table 2. Solutions based on varying initial guesses of τ_c and k_d for the Blaisdell solution and the scour depth solution.

		Barren Fork Creek Site 1				Barren Fork Creek Site 2			
Initial Guess		Blaisdell Solution		Scour Depth Solution		Blaisdell Solution		Scour Depth Solution	
τ_c (Pa)	k_d ($cm^3 N^{-1} s^{-1}$)	τ_c (Pa)	k_d ($cm^3 N^{-1} s^{-1}$)	τ_c (Pa)	k_d ($cm^3 N^{-1} s^{-1}$)	τ_c (Pa)	k_d ($cm^3 N^{-1} s^{-1}$)	τ_c (Pa)	k_d ($cm^3 N^{-1} s^{-1}$)
0.01	73.1	0.1	23.3	1.4	121.2	1.3	Error	14.3	6.6
0.1	10.8	0.1	23.3	1.4	121.2	1.3	Error	14.3	6.6
1	1.6	0.1	23.3	1.4	121.2	1.3	1.1	14.3	6.6
10	0.2	0.1	23.3	1.4	121.2	1.3	1.1	14.3	6.6

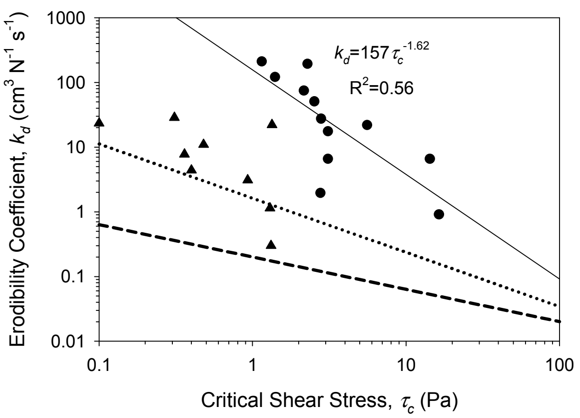


Figure 7. Correlation between k_d and τ_c (solid line) for the Illinois River watershed JET tests (triangles are derived from the Blaisdell solution, and circles are derived from the scour depth solution) and comparison to previously proposed relationships by Hanson and Simon (2001) (dashed line) and Simon et al. (2011) (dotted line).

SOFTWARE AVAILABILITY

A copy of the new spreadsheet tool is available upon request. Requests may be sent to the corresponding author, Dr. Garey Fox, at garey.fox@okstate.edu.

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