BAE 820 Physical Principles of Environmental Systems

Turbulent transfer and log wind profile

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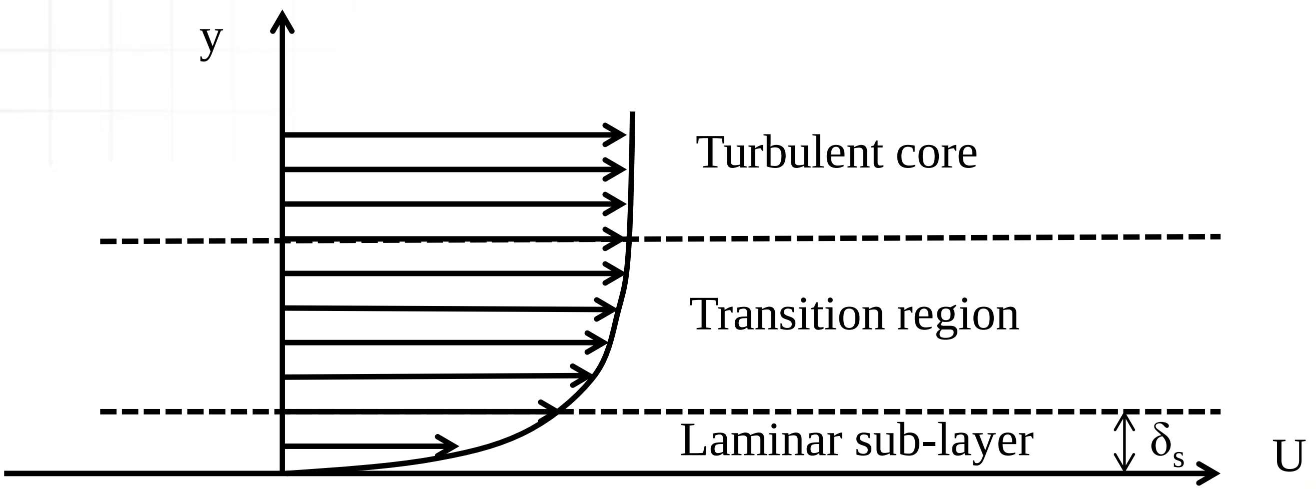
Turbulent (eddy) diffusion

- Until now we have considered mass transfer by molecular diffusion. If turbulent conditions prevail there will be an additional transport contribution by the turbulent eddies.
- In contrast to molecular diffusivity, which is considered to be properties of the fluid, turbulent diffusivity depends on nature of turbulence.
- In contrast to molecular diffusion, which is due to fluid motion, not molecular motion, turbulent diffusion is due to fluid motion.
- Turbulent or eddy diffusivity E_D has been introduced and it is structurally similar to molecular diffusivity D. The general transfer equation can be written to include both molecular and eddy diffusivities.

$$J = -(D+E_D)\frac{\partial c}{\partial y} = K(C_0-C_b)$$

Flow regimes near a surface

- The relative magnitude of the molecular and eddy diffusivities depends on the position of the fluid relative to the wall. In a thin region very close to the wall, no turbulence is present. This region is called the laminar sub-layer. The region far from the wall is called the turbulent core, where eddy diffusivities are much larger than molecular diffusivities. The region between the turbulent core and the laminar sub-layer is called as the transition region or buffer layer, where both molecular and eddy diffusivities are important. As the turbulence level increases, the thickness of the laminar sub-layer δ_s decreases. In general, $\delta_s \sim (1/U\infty)$.



The friction velocity

- Between the wall and the free stream the velocity varies over the vertical coordinate. The velocity gradient is called shear.
- Turbulence is an instability generated by shear. The turbulence level scales on the shear. So we introduce friction velocity (shear velocity) U* scale to represent the shear strength. It characterizes the shear at the boundary.

$$U^* = (\tau/\rho)^{0.5} = \left(\frac{\eta}{\rho}\frac{\partial U}{\partial y}\Big|_{y=0}\right)^{0.5} = \left(\nu\frac{\partial U}{\partial y}\Big|_{y=0}\right)^{0.5}$$

Where τ is shear stress (N/m^2), ρ is fluid density, and ν is kinetic viscosity of the fluid.

The law of the wall

- Define dimensionless terms

$$U^+=U/U^*,\ y^+=yU^*/\nu$$

For $y^+<5$	Laminar sub-layer	$U^+=y^+$	$U(y) = yU^{*2}/\nu$
For $5<y^+<30$	Buffer layer	$U^+=5.0\ln y^+-3.05$	
For $y^+>30$	Turbulent core	$U^+=2.5\ln y^+ +5.0$	$U(y) = \frac{U*}{\kappa}\ln(y/y_0)$

κ = 0.4~0.42 is an empirical constant, known as the von Karman's constant.
y_0 is characteristics roughness of the surface. It is the distance from the surface at which the idealized velocity given by the law of the wall goes to zero.

The concentration profile

- For the laminar sub-layer ($y^+<5$),

$$C_0\text{-}C_5 = \frac{5\nu J}{U{*}D} = \frac{5J}{U{*}}\, S_c$$

- For the buffer layer ($5<y^+<30$)

$$C_5\text{-}C_{30} = \frac{5J}{U{*}} \ln(5Sc+1)$$

- For the turbulent core ($y^+>30$)

$$C_{30}\text{-}C_b = \frac{J}{U{*}}\left[\frac{U_b}{U{*}} - 5(\ln 6+1)\right]$$

Thus the total concentration difference is

$$C_0\text{-}C_b = \frac{J}{U{*}}\left[5S_c + 5\ln(5Sc+1) + \frac{U_b}{U{*}} - 5(\ln 6+1)\right]$$

- Thus,

$$K = \frac{J}{C_0 - Cb} = \frac{U{*}}{5S_c + 5\ln(5Sc+1) + \frac{U_b}{U{*}} - 5(\ln 6+1)}$$

The von Karman analogy

- The friction factor f is defined as in

$$\tau = f\rho U^2/2$$

Where τ is the shear stress.

- f can be related with U* as in

$$f/2 = \tau/(\rho U^2) = (U^*/U)^2$$

- Thus we obtain the von Karman analogy

$$\frac{K}{U_b} = \frac{f/2}{1+5\sqrt{f/2}\left[S_c+5\ln\left(\frac{5Sc+1}{6}\right)-1\right]}$$

- For S_c of around1, the von Karman analogy reduces to Reynolds analogy.

$$\frac{K}{U_b} S_c^{2/3} = \frac{S_h}{R_e S_c^{1/3}} = \frac{f}{2}$$

Several empirical equations

- For turbulent flow in a smooth tube

$$\frac{S_h}{R_e S_c^{1/3}} = \frac{f}{2} = 0.023R_e^{-0.2}$$

- For single spheres

$$S_h=2+0.552R_e0.5S_c^{1/3}$$

- For packed beds of granular solids (R_e<50)

$$\frac{S_h}{R_e S_c^{1/3}} = \frac{f}{2} = 1.82R_e^{-0.51}$$

- For laminar flow across a flat plate

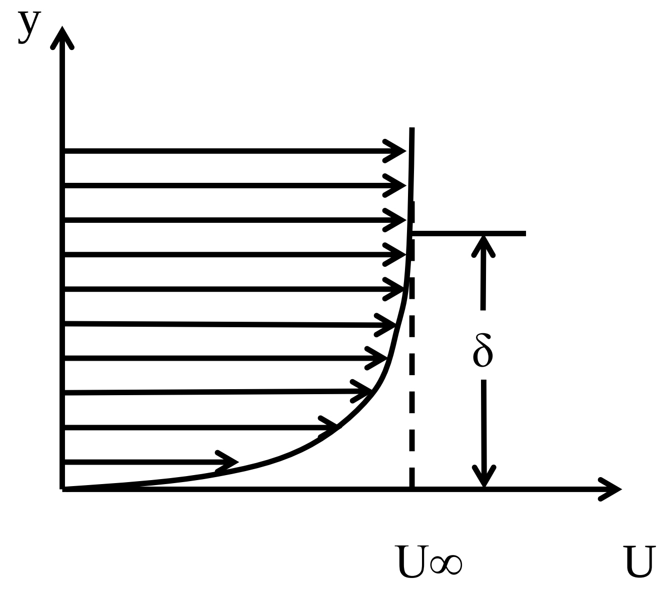
$$\frac{S_h}{R_e S_c^{1/3}} = \frac{f}{2} = \frac{1.33}{2R_e^{1/2}}$$

- For turbulent flow across a flat plate

$$\frac{S_h}{R_e S_c^{1/3}} = \frac{f}{2} = \frac{0.074}{2R_e^{0.2}}$$

The turbulent boundary layer

- Between the wall and the free stream the velocity varies over the vertical coordinate.
- The velocity gradient is called shear.
- The region of velocity shear near the wall is called the momentum boundary layer. The height of the boundary layer, δ, is typically defined as the distance above the bed at which u = 0.99 U∞.



The log wind profile

- In the atmosphere, over an open, level, and relatively smooth surface, wind speed increases logarithmically with height as in the law of the wall.

$$U(z) = \frac{U_*}{\kappa} \ln(z/z_0)$$

Where κ is the von Karman's constant (κ = 0.4~0.42). z_0 is characteristics roughness of the surface. It can be empirically expressed as

$$\log_{10} z_0 = 0.997 \log_{10} h - 0.883$$

Where h is crop height, and both z_0 and h are expressed in m.

- A modified equation is

$$U(z) = \frac{U_*}{\kappa} \ln\left(\frac{z-d}{z_0}\right)$$

Where d is the zero plane displacement height. It may be assumed to be 0.5 to 0.8 of the crop height, h,.

The log wind profile

With knowledge of the wind speed profile

- Effectiveness of the vertical exchange processes can be estimated. (i.e., friction velocity U* or shear stress τ can be calculated).
- Based on known wind speed at reference height, wind speed at some other height can be estimated using the following equation.

$$\frac{U_2}{U_1} = \frac{\ln(z_2 - d) - ln z_0}{\ln(z_1 - d) - ln z_0}$$

Eddy diffusivity in the atmosphere

- A general eddy diffusion equation is expressed as

$$J_D = -E_D\frac{\partial c}{\partial z}$$

- In the lower atmosphere, vertical transport of gases is due to relative movement of parcels of air from one level to another resulting eddy motion. The eddy diffusivity can be estimated by

$$E_D = \kappa U^*z/\varphi$$

Where κ is the von Karman's constant. U* is friction velocity, z is the height at which E_D is been estimated, and φ is the stability function for gas transport and is derived experimentally and is based on the Richardson number R_i, which can be calculated from gradient of temperature and wind speed. It is a dimensionless number which is positive under stable condition, and negative under unstable condition, and approach zero under neutral condition. $R_i = \frac{g\left(\frac{\partial \theta}{\partial z}\right)}{T\left(\frac{\partial u}{\partial z}\right)^2}$

  - For neutral condition: $\varphi = 1$
  - For stable condition: $\varphi = (1-5R_i)^{-1}$
  - For unstable condition: $\varphi = (1-16R_i)^{-1/2}$

- When wind speed is zero, E_D will be zero, but there will still be some gas transport due to molecular diffusion and thermal buoyancy.

A micrometeorology method to estimate gas flux from land or liquid surfaces

- The integrated horizontal flux method: assume surface emission from a upwind plot transported horizontally by eddy wind movement is captured in a vertical plane downwind. A simplified expression is

$$J=\frac{1}{x}\int_{z_0}^{z_p} UC\,dz$$

Where

- x is the distance that the wind has traveled over the surface,
- U and C are mean horizontal wind speed and mean gas concentration at a height in the vertical plane,
- z is the height increment,
- z_0 is the height at which wind speed is zero,
- z_p is the height at which gas concentration approaches background levels.

Example

- Given the following measurements of wind speed and NH_3 concentrations at center of a round plot with r=32m, estimate gas flux from the plot.

Height(m)	Δz (m)	Mean air velocity (m/s)	NH_3 concentration ($\mu g/m^3$)	$U*C*\Delta z$ ($\mu g m^{-1} s^{-1}$)
3.4	1.2	1.9	331	754
2.2	1.2	1.62	313	608
1.2	0.8	1.24	630	625
0.6	0.4	0.89	978	348
0.2	0.4	0.34	1836	250

Thus, using the integrated horizontal flux method

$$J = 1/x\ \Sigma(UC\Delta z) = 1/32 \times 2584 = 80.75\ \mu g/(m^2 s)$$

Units for atmospheric Species

- Concentration:
  - The amount (or mass) of a substance in a given volume divided by that volume.
  - Expressed as mole/m^3, μg/m^3, …

- Mixing ratio:
  - The ratio of the amount (or mass) of the substance in a given volume to the total amount (or mass) of all constitutes in that volume.
  - Expressed as ppm, ppb, …