BAE 820 Physical Principles of Environmental Systems

Particle dynamics

Dr. Zifei Liu

K·STATE Research and Extension | Biological and Agricultural Engineering

Straight-line particle acceleration

In the previous sections, we discussed the terminal settling velocity under the equilibrium condition (i.e. the net force acting on the particle is zero and the velocity of the particle is constant). Now, we are going to consider the particle during the acceleration process.

If a particle is released with zero initial velocity in still air. How long will it take for the particle to reach its terminal settling velocity?

Consider for the moment the motion of a particle in a fluid in the presence only of gravity. The acceleration of the particle can be expressed as

$$\frac{4}{3}\pi r^3 \rho_p \frac{dV}{dt} = \frac{4}{3}\pi r^3 (\rho_p - \rho_l) g - F_d$$

Straight-line particle acceleration

In laminar flow,

$$\frac{4}{3}\pi r^3\rho_p\frac{dV}{dt} = \frac{4}{3}\pi r^3(\rho_p-\rho_l)g - 6\pi r\eta v/C_c$$

In most cases, $\rho_p >> \rho_l$, ρ_l can be neglected, divide the equation by $\xi=6\pi r\eta/C_c$, we obtain

$$\frac{4r^2\rho_p C_c}{18\eta}\frac{dV}{dt} = \frac{4r^2\rho_p C_c}{18\eta}g - v$$

Define $\tau = \frac{4r^2\rho_p C_c}{18\eta} = \frac{D_p{}^2\rho_p C_c}{18\eta}$,

The equation can be written as

$$\tau\frac{dV}{dt} = \tau g - v$$

Straight-line particle acceleration

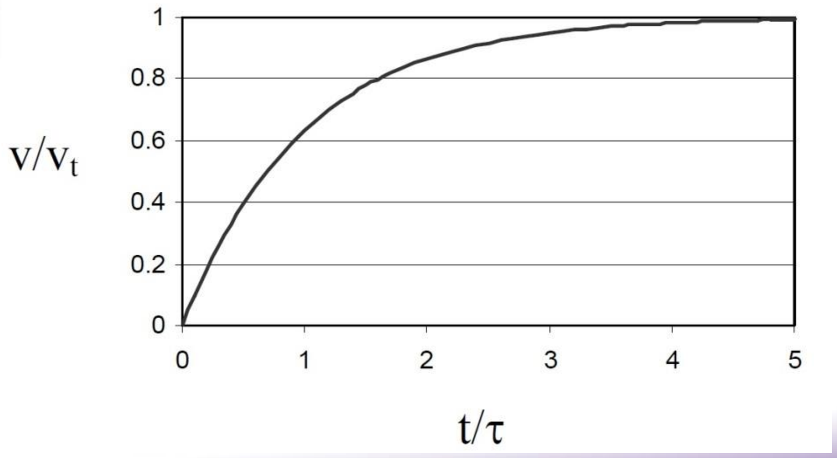
If the particle is at rest at t = 0, the equation can be solved as

$$v(t) = \tau g[1 - \exp(-\frac{t}{\tau})]$$

For t>> τ, the particle attains a constant velocity (terminal settling velocity)

$$v_t = \tau g = \frac{D_p{}^2 \rho_p g C_c}{18\eta}$$

A particle will reach 63% of its terminal settling velocity after an elapsed time of τ, and reach 95% of the terminal settling velocity after 3τ.



Particle relaxation time

The term $\tau = \frac{D_p{}^2 \rho_p C_c}{18\eta}$ is called relaxation time. It is the characteristic time for the particle to approach steady motion. It characterizes the time required for a particle to adjust or "relax" its velocity to a new condition of forces. It's an indication of the particle's ability to quickly adjust to a new environment or condition. It depends on the mass and mechanical mobility of the particle, and is not affected by the external forces acting on the particle.

$$\tau = V_t/g = \text{mass}\times\text{mobility} = \frac{4}{3}\pi r^3 \rho_p \times \frac{C_c}{6\pi r \eta}$$

Because relaxation time is proportional to the square of particle diameter, it increases rapidly with the increase of particle size. Usually, small particles "relax" to new environments (i.e. following the flow well) in a very short time, while larger particles are more "stubborn" and tend to stick to their original path.

Magnitude of particle relaxation time

If a particle enters a moving airstream, it approaches the velocity of the stream with the characteristic relaxation time τ.

The characteristic time for most particles of interest to achieve steady motion in air is extremely short. The velocity of a particle in a fluid very quickly adjusts to a steady state at which the drag force is balanced by the sum of the other forces acting on the particle.

Relaxation time for unit density particle in the air (p=1 atm, T=293 K)

D_p (μm)	τ (s)	v_t (m/s)	Stopping distance (v_0=1m/s)	Stopping distance (v_0=10m/s)
10	3.1×10^{-4}	3.0×10^{-3}	310μm	2.3mm
1	3.6×10^{-6}	3.5×10^{-5}	3.6μm	36μm
0.1	9.0×10^{-8}	8.8×10^{-7}	0.09μm	0.9μm
0.01	7.0×10^{-9}	6.9×10^{-8}	0.007μm	0.07μm

Traveling with an initial velocity

For particles traveling in the still air with an initial velocity v_0 and without any external forces (i.e. only drag force is acting on the particle), we can get its velocity equation as

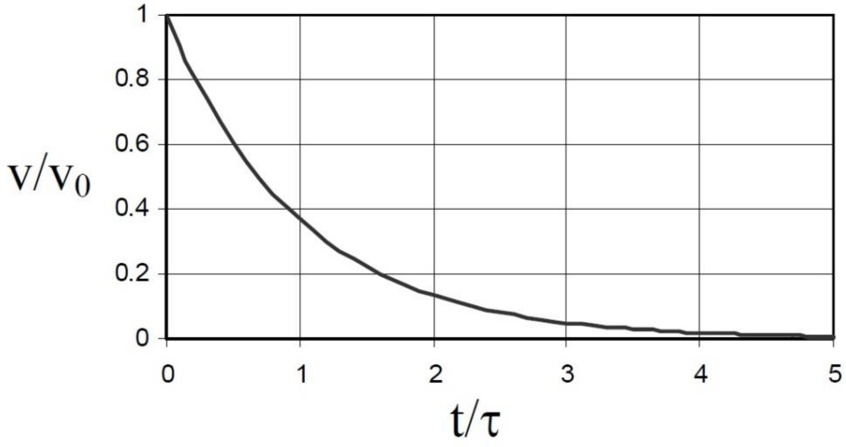
$$\frac{4}{3}\pi r^3 \rho_p \frac{dV}{dt} = -F_d = -6\pi r \eta v / C_c$$

$$\frac{4r^2\rho_p C_c}{18\eta}\frac{dV}{dt} = \tau\frac{dV}{dt} = -v$$

Solve the equation, we get

$$v(t) = v_0 \exp\left(-\frac{t}{\tau}\right)$$

$$x(t) = \tau v_0\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$$



Stopping distance

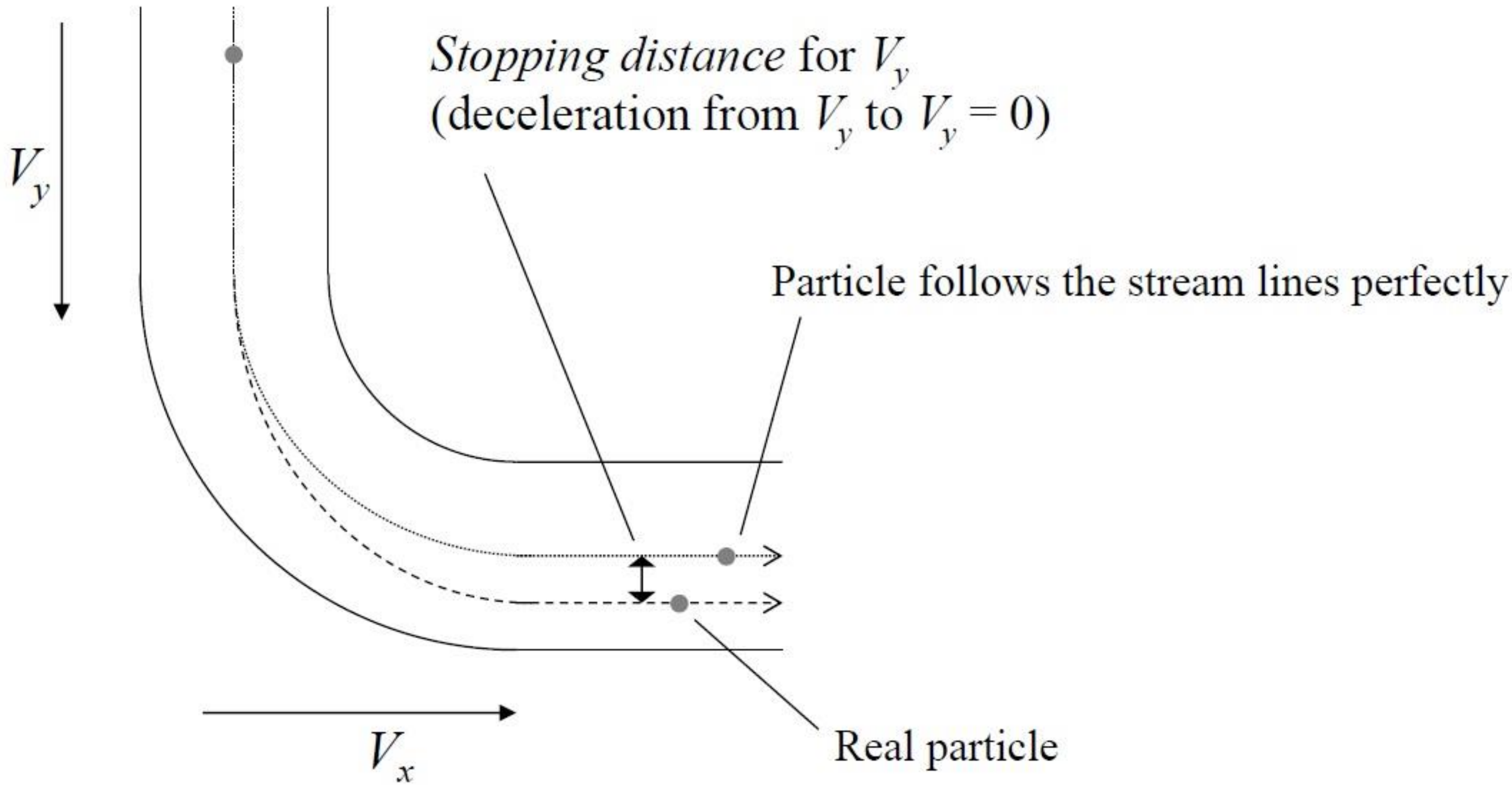
The maximum distance a particle with an initial velocity will travel in the still air without any external forces is defined as stopping distance. Similar to relaxation time, stopping distance indicates the ability of a particle to respond to a new condition.

Taking the condition that at $t = \infty$, we can get the formula for the stopping distance.

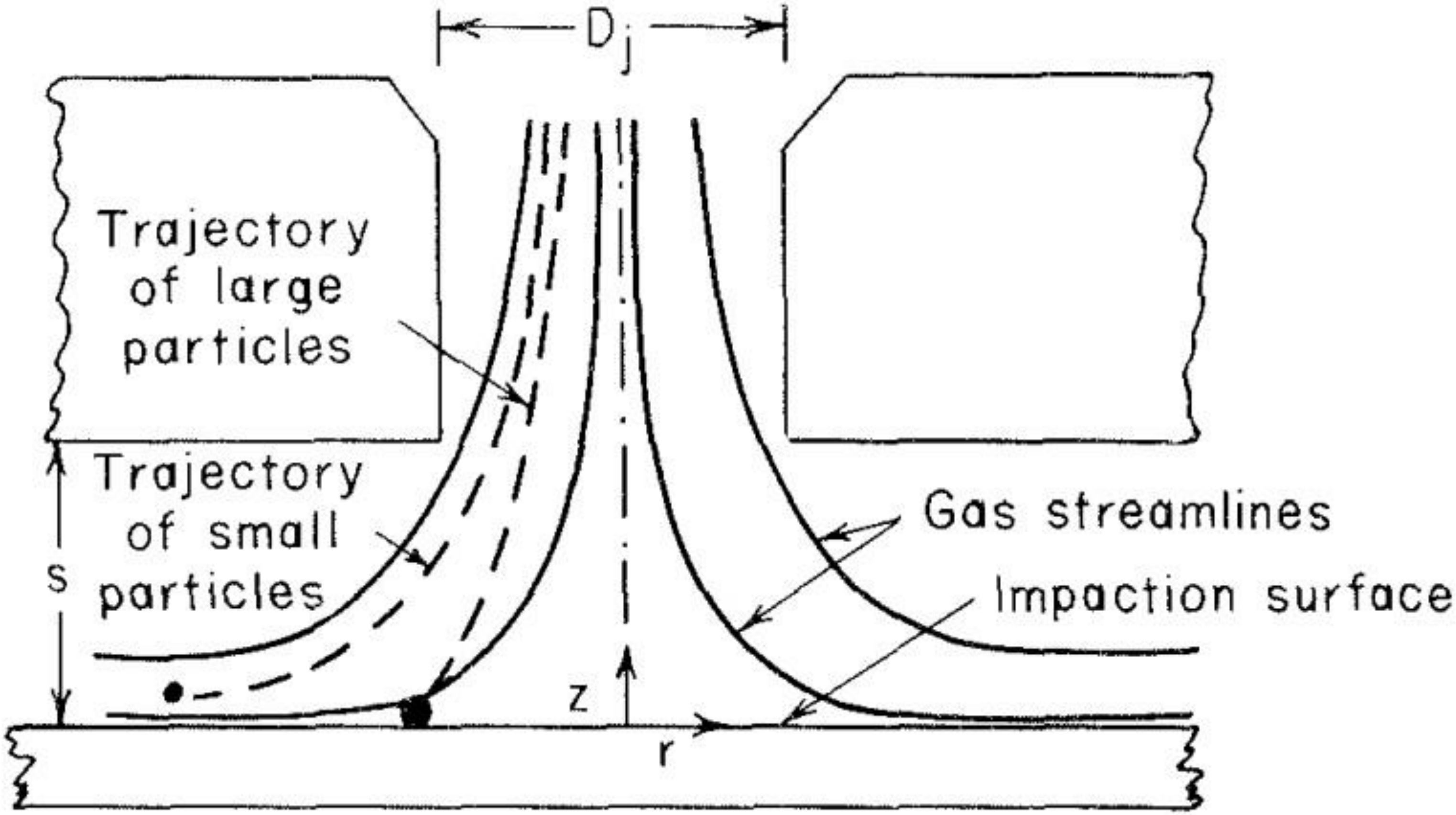
$$S = \tau v_0 = v_0 \frac{D_p^2 \rho_p C_c}{18\eta}$$

For a l μm-diameter unit density spheres , with an initial speed of 10 m/s, the stop distance is 3.6×10^{-3} cm.

Curvilinear particle motion



Particle trajectories on a cascade impactor



The Stokes number

The Stokes number (Stk) is the ratio of the stop distance to a characteristic length scale of the flow. As particle mass decreases, Stk decreases. A small Stk implies that a particle adopts the fluid velocity very quickly. In a sense, Stk can be considered as a measure of the inertia of the particle. Equality of Stk between two geometrically similar flows indicates similitude of the particle trajectories.

$$Stk = \tau v_0/L = v_0 \frac{D_p{}^2 \rho_p C_c}{18 \eta L}$$

When outside of Stokes region

Above equation is valid only for Stokes region. When $R_e>1$, the stopping distance is shorter than is predicted by the equation since drag force increases as the R_e increases. This increase in drag force attenuates the velocity of the particle, and hence reduces the stopping distance.

Since the R_e is proportional to the particle velocity and the drag force is proportional to the V^2, it is extremely difficult to obtain stopping distance outside of Stokes region. Mercer (1973) proposed an emprical equation within 3% accuracy to calculate stopping distance. For particles having an initial $R_{e0}<1500$.

$$S = \frac{\rho_p d}{\rho_g}\left[\mathrm{Re}_0^{1/3} - \sqrt{6}\,\arctan\left(\frac{\mathrm{Re}_0^{1/3}}{\sqrt{6}}\right)\right]$$

Particle diameter

Equivalent volume diameter (D_e)	Diameter of a sphere that would have the same volume and density as the particle	Only standardizes the shape of the particle by its equivalent spherical volume
Stokes diameter (D_s):	Diameter of the sphere that would have the same density and settling velocity as the particle	Standardizes the settling velocity of the particle but not the density
Aerodynamic diameter (D_a):	Diameter of a spherical particle with standard density $\rho_0 = 1$ g/cm^3 which has the same terminal settling velocity in air as the particle of interest	Standardizes both the settling velocity and the particle density.

Dynamic shape factor

- Most particle in practice are nonspherical. Particle dynamic shape factor χ is defined as the ratio of the actual resistance force of a nonspherical particle to the resistance force of a spherical particle that has the same equivalent volume diameter (d_e) and the same settling velocity as the nonspherical particle.

$$F_D = \frac{3\pi D_e \eta v}{C_{de}} \chi$$

- Where F_D is the actual drag force exerted on the nonspherical particle and C_{de} is the slippage correction factor for d_e. The dynamic shape factor is always greater than 1 except for certain streamlined shapes .

Shape	Dynamic shape factor, χ
Sphere	1.00
Cube	1.08
Fiber	1.06
Dust	
Quartz	1.36
Sand	1.57

Aerodynamic diameter

$$V_t = \frac{D_a{}^2 \rho_0 g C_{ca}}{18\eta} = \frac{D_e{}^2 \rho_p g C_{ce}}{18\eta\chi}$$

So,

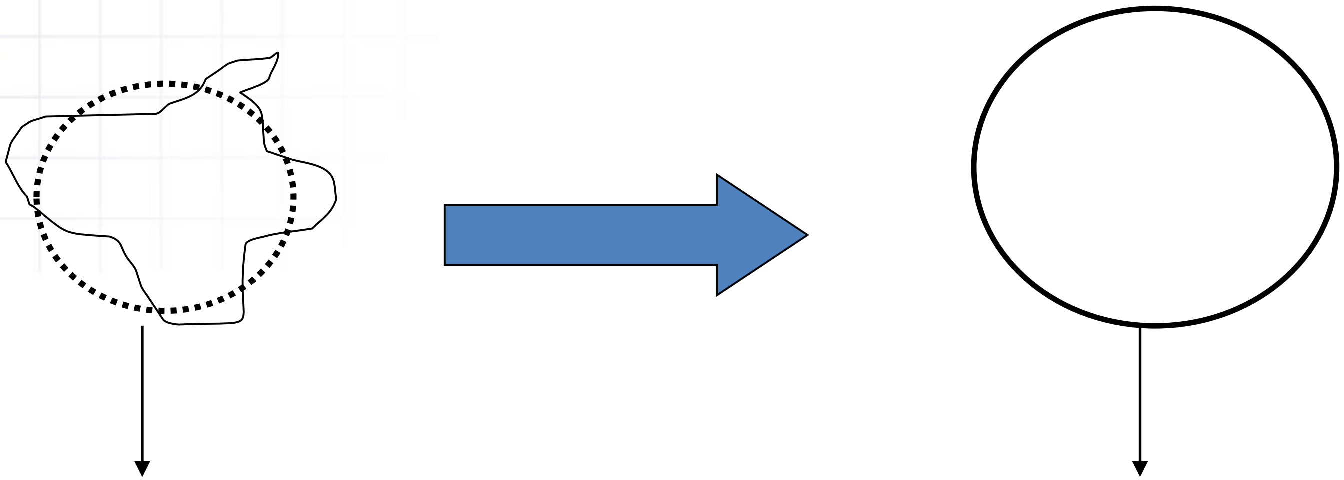
$$D_a = D_e\left(\frac{\rho_p C_{ce}}{\rho_0 \chi C_{ca}}\right)^{0.5}$$

D_a standardizes both the settling velocity and the particle density. Thus it is a convenient variable to use to analyze particle behavior and design of particle control equipment.

Aerodynamic diameter

- An irregular particle
- D_e =10 μm
- ρ_p = 3000 kg/m^3
- χ =1.3

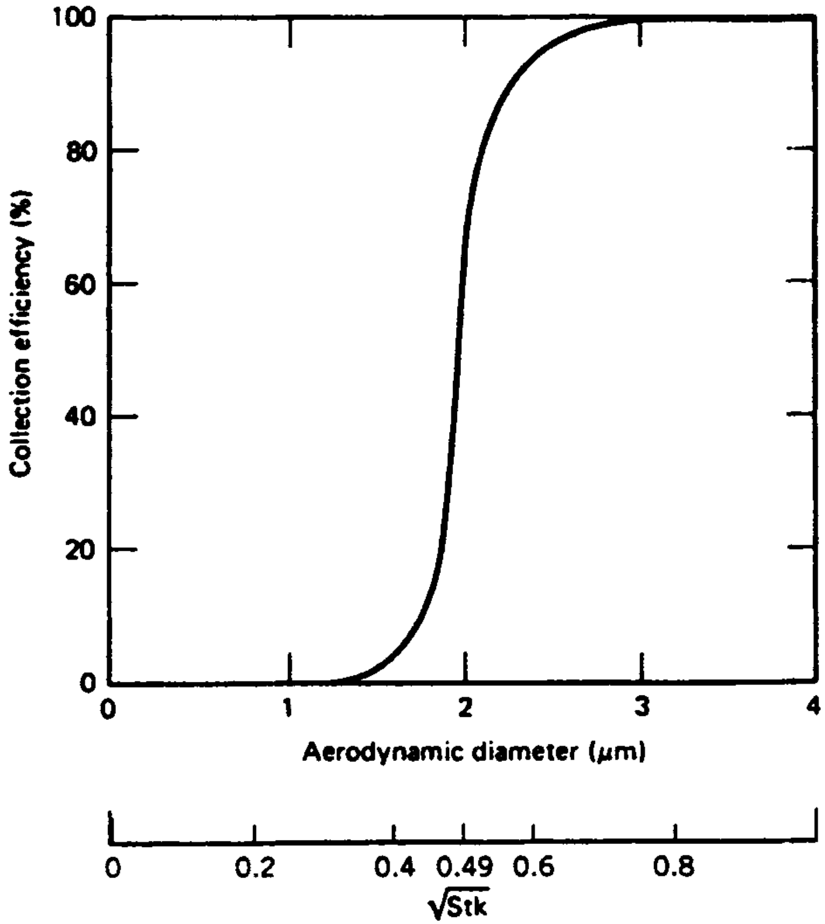
- The aerodynamic equivalent sphere
- D_a=15.2 μm
- ρ_p=1000 kg/m^3



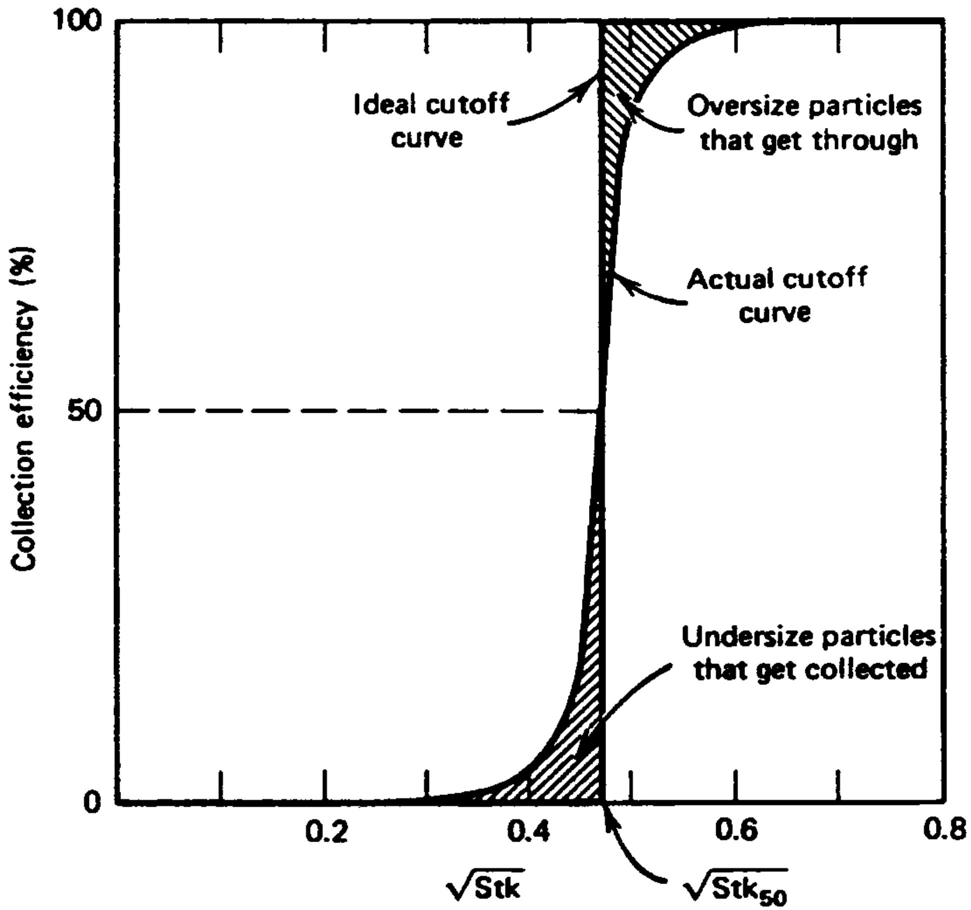
v_t=0.0084 m/s

v_t=0.0084 m/s

Collection efficiency characteristics of an impactor



Size selective sampling: ideal vs. real



Concept of cutoff size

The impaction collection efficiency is a function of the Stokes number, and it increases as Stk increases. Their efficiency curves are characterized by Stk_{50} (50% collection efficiency) meaning that the mass of particles below cut-off size collected equals the mass of particles getting through larger than the cut-off. When the Stokes number that gives 50% collection efficiency, the size of particles is called the cutoff size (D_{p50}).

For the impactor meeting recommended design criteria, the Stokes number for 50% collection efficiency (Stk_{50}) is 0.24 for circular jets and 0.59 for rectangular jets.

$$Stk_{50} = v_0 \frac{(D_{p50})^2 \rho_p C_c}{18\eta L} = v_0 \frac{(D_{p50})^2 \rho_p C_c}{18\eta D_j/2}$$

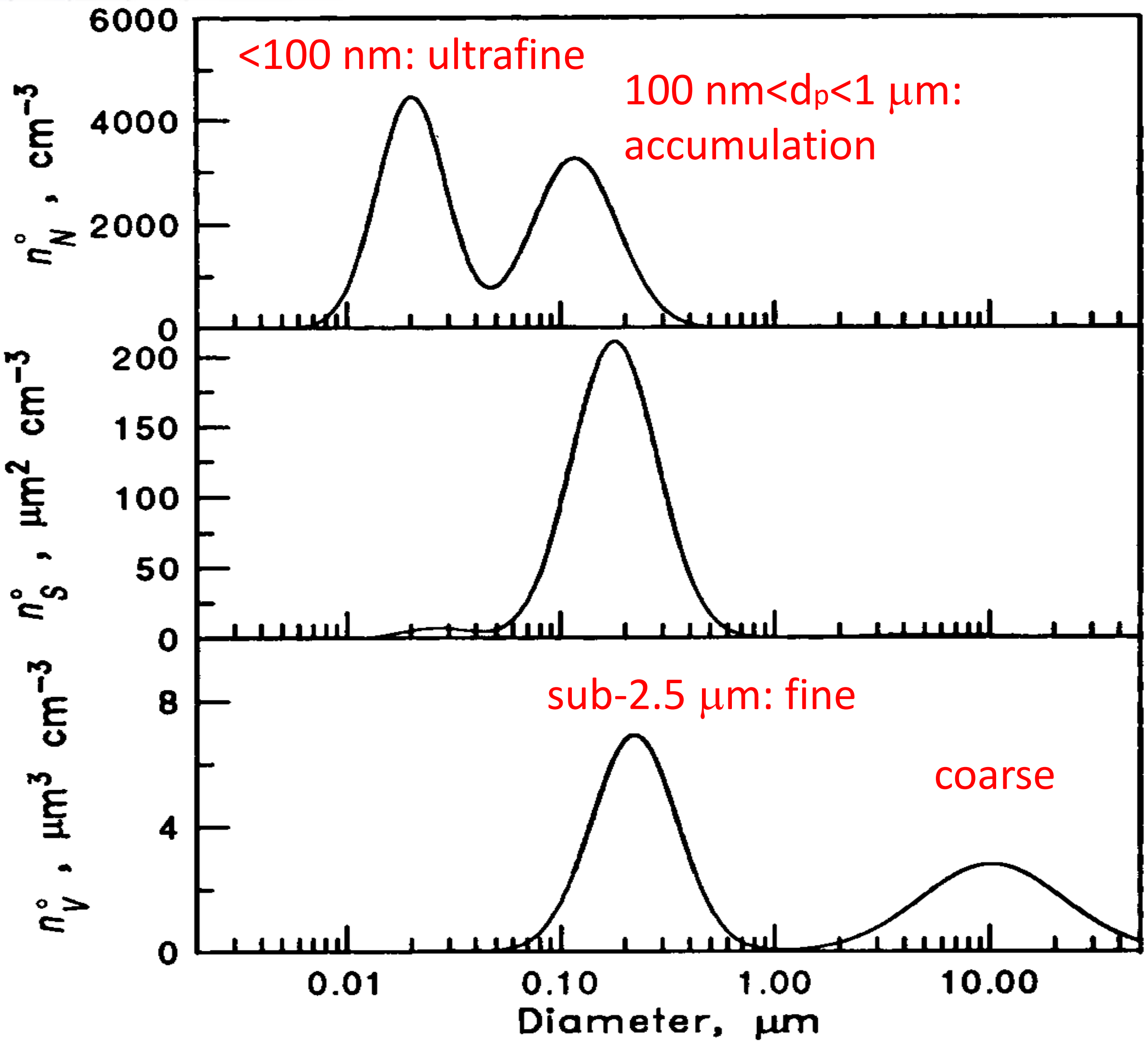
$$D_{p50}\sqrt{C_c} = \left(\frac{9\eta D_j Stk_{50}}{\rho_p v_0}\right)^{0.5} = \left(\frac{9\pi \eta D_j{}^3 Stk_{50}}{4\rho_p Q}\right)^{0.5}$$

Typical size distribution of aerosol in air

Number
- Cloud formation

Surface area
- Visibility

Volume
- Mass
- Human health



Problem solving #3

A 1 μm diameter spherical particle with specific gravity 2.0 (ρ_p=2000kg/m^3) is ejected from a gun into standard air at a velocity of 10 m/s.

How far does it travel before it is stopped by viscous friction? What is its relaxation time? What is its terminal settling velocity?

Problem Solution #3

Check $R_e = D_p \rho_l v_0/\eta = \dfrac{1\times10^{-6}\times1.2\times10}{1.8\times10^{-5}} = 0.67 < 1$

$$\tau = \frac{D_p{}^2\rho_p C_c}{18\eta} = \frac{(1\times10^{-6})^2\times2000\times1.17}{18\times1.8\times10^{-5}} = 7.2\times10^{-6}\ \text{s}$$

$$S = v_0\,\tau = 10\times7.2\times10^{-6} = 7.2\times10^{-5}\ \text{m} = 72\ \mu\text{m}$$

$$v_t = \tau g = 72\times10^{-6}\times9.8 = 7\times10^{-4}\ \text{m/s}$$